**Lecture Note-6**

**Complex Integration**

**Line integral in the complex plane**

Complexdefinite integrals are called (complex) **line integrals**. They are written as

.

Here the **integrand**is integrated over a given curve *C.* This curve *C* in the complex plane is

called the **path of integration**.

If *C* is a **closed path** (one whose terminal point coincides with its initial point),

then it is denoted by .

**Partitioning of path *C*:** If *C* is a combination of *C*1 and *C*2 then, 

We may represent Cby a parametric representation . That is, . The sense of increasing *t* is called the **positive sense** on *C.*

**Note:** Parametric representation of any curve is not unique.

**Example 2:** Find and sketch the path whose orientation is given by .

|  |  |
| --- | --- |
| **Solution:**      Comparing real and imaginary part,  we get , .  So, represents upper semicircle of radius 2 with center (0,0). | **C:\Users\aiub\Desktop\1.png**  **Fig: 2** |

**Lecture Note-7**

**Integration using Cauchy’s Residue** **Theorem (CRT)**

Two main reasons account for the importance of integration in the complex plane. The practical reason is that complex integration can evaluate certain real integrals appearing in applications that are not accessible by real integral calculus. The theoretical reason is that some basic properties of analytic functions are difficult to prove by other methods. Complex integration also plays an important role in connections with special function, such as the gamma function, the error function, various polynomials and others, and the application of these functions in physics.

**Cauchy’s Integral Formula:**

If a function  is analytic within and on a simple closed contour  and if  is any point interior to then,

**- - - - - - - - - (1)**

**Special case:** If is not an interior point of the contour  then **.**

Differentiating n-1 times w.r.to

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**Definition of singular point (of an analytic function):**

A point at which an analytic function is not defined, i.e., at which fails to exist, called a singular point or pole or singularity of the function.

**Example 7.1:** If then , 3 are the singular points of 

**Residue Finding Method:**

If is analytic inside and on a simple closed curve *C* except at pole or has singularity at  of order 1, then

.

If is analytic inside and on a simple closed curve *C* except at pole or has singularity at of order ***m***, then



**Cauchy Residue Theorem:**

If is analytic inside and on a simple closed curve *C* except at a finite number of singular points  inside *C*, then

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**Example 7.2:** Evaluate by CRT , where C is the circle 

**Solution**: For singular point,



Singular point is a pole of order 2. The point lies inside the circle .

Residue at the point  is,